Simulation Based Conductivity Tensor Determination of Sintered Nanosilver

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Introduction

3D printing of conductive and dielectric materials in one process is an emerging technology. Modern 3D printers like the Nano Dimension's Dragonfly LDM® are able to print real 3D structures such as coils, antennas or capacitors. The design of the devices requires reliable material parameters, especially the electrical conductivity is important for electromagnetic applications. The printed structures show a print direction dependent pattern (see figure 2) and an anisotropic conductivity. In addition the conductivity can depend on the position on the printing pad and other influences [1]. The determination of the conductivity, for example with the Montgomery Method [2], is based on a measurement of the resistance of test structures. Materials with good conductivity require large samples to generate a significant voltage drop but large samples limit the local resolution. Here we propose an approach based on FEM simulation in combination with optimization to determine the conductivity.

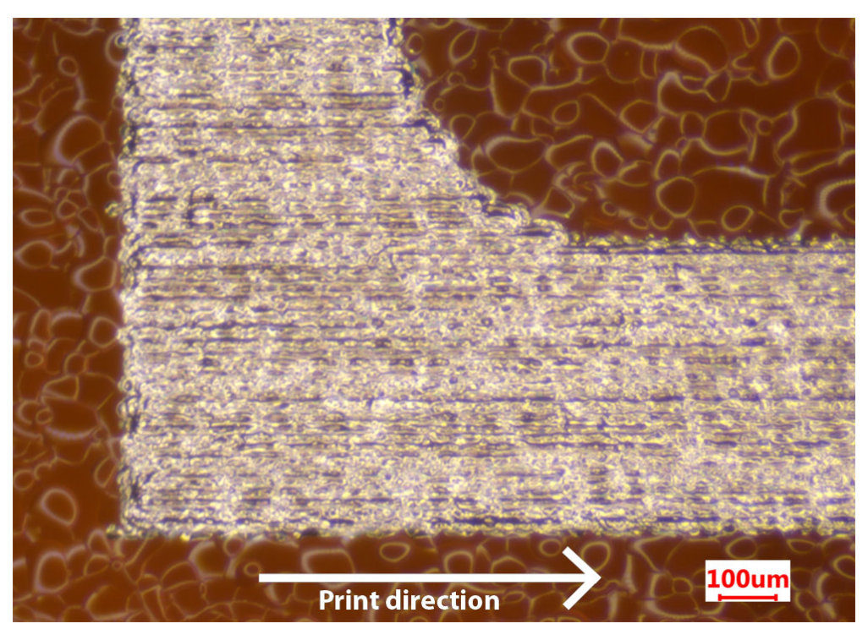


Figure 2: Part of a printed electric circuit with visible printing direction.

Anisotropic Conductivity

For many materials the electrical conductivity σ in Ohm's law can be reduced to a scalar value. For complex materials it is a 3×3-Tensor. If the area under consideration includes many unit cells and no voltages are observed perpendicular to the current, as for example none of those occurring with the Hall effect, the conductivity simplifies to a diagonal matrix [3].

$$\vec{J} = \sigma \vec{E} \tag{1}$$

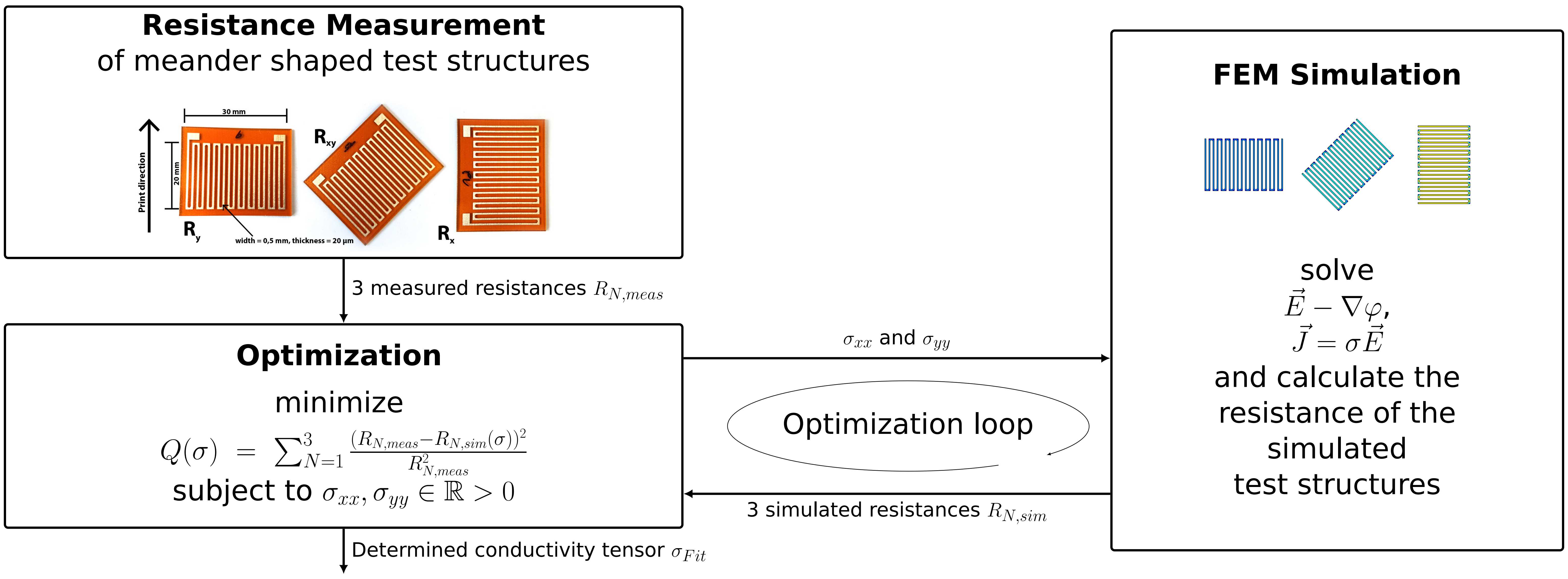
$$\sigma = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zy} & \sigma_{zy} & \sigma_{zz} \end{pmatrix} \Rightarrow \sigma = \begin{pmatrix} \sigma_{xx} & 0 & 0 \\ 0 & \sigma_{yy} & 0 \\ 0 & 0 & \sigma_{zz} \end{pmatrix} \tag{2}$$

The diagonal matrix as conductivity tensor is assumed for the examined printed material.

Method for the Determination of Electrical Conductivity

The test structures are meander-shaped in order to achieve a long conductor length on a small area. To determine the conductivity tensor we use a combination of optimization (MATLAB®) of FEM simulation (COMSOL Multiphysics®). The conductivity tensor with the free parameters σ_{xx} and σ_{yy} is used to fit the simulation to the measured resistances.

The component σ_{zz} cannot be determined due to the planar geometry. As optimization criterion we use the normalized least-squares between measured and simulated resistance. The optimization algorithm is a Nelder-Mead-Method [5] with a transformation to consider boundaries for the free parameters [6].



Resistance Measurement
of meander shaped test structures

3 measured resistances $R_{N,meas}$

Optimization
minimize
$$Q(\sigma) = \sum_{N=1}^{3} \frac{(R_{N,meas} - R_{N,sim}(\sigma))^2}{R_{N,meas}^2}$$
subject to $\sigma_{xx}, \sigma_{yy} \in \mathbb{R} > 0$

σ_{xx} and σ_{yy}

Optimization loop

3 simulated resistances $R_{N,sim}$

FEM Simulation

solve
$\vec{E} - \nabla\varphi$,
$\vec{J} = \sigma\vec{E}$
and calculate the resistance of the simulated test structures

Determined conductivity tensor σ_{Fit}

Results

The determined conductivity tensor is shown in (3). The tensor shows the expected anisotropy. The meander simulation at the end of the optimization shows a good agreement with the measurement (see table 3).

$$\sigma_{Fit} = \begin{pmatrix} 1.41 & 0 & 0 \\ 0 & 3.71 & 0 \\ 0 & 0 & \sigma_{zz} \end{pmatrix} \frac{MS}{m} \tag{3}$$

Figure 3: Measured and simulated resistance values of the meander test structures.

	Measured	Simulated
R_x/Ω	27.9	27.6
R_{xy}/Ω	19.7	19.9
R_y/Ω	11.8	11.7

To investigate the transferability of the values to other structures, a further test structure of lines (see figure 5) is printed. Figure 7 shows the measured and simulated resistance values of the eleven lines denoted by their angle ϕ. In the printing direction $\phi = 90°$ the simulated and measured resistance values are similar. Perpendicular to the printing direction $\phi = 0°$ the resistance value of the simulation differs from the experimental value. One explanation for this is the possible position dependence of the conductivity. Further influences of the print process on the conductivity are to be expected [1].

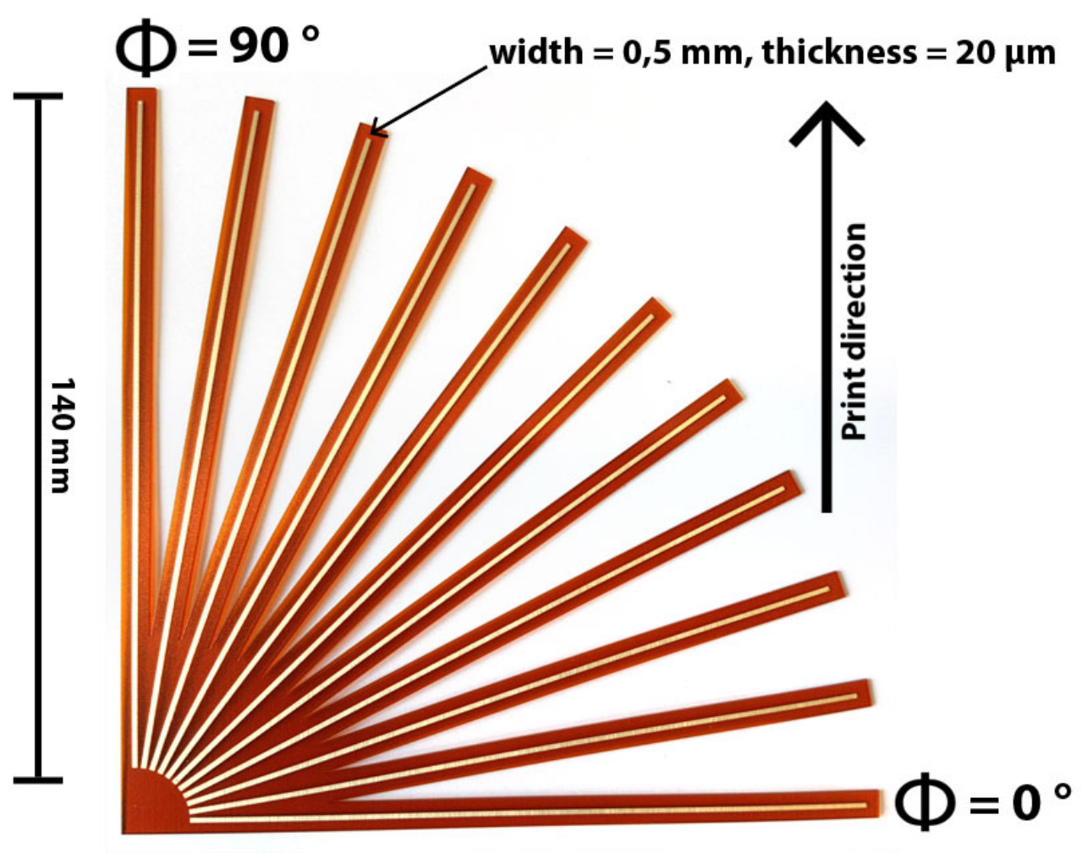


Figure 5: Test structure to verify the transferability of the determined conductivity values.

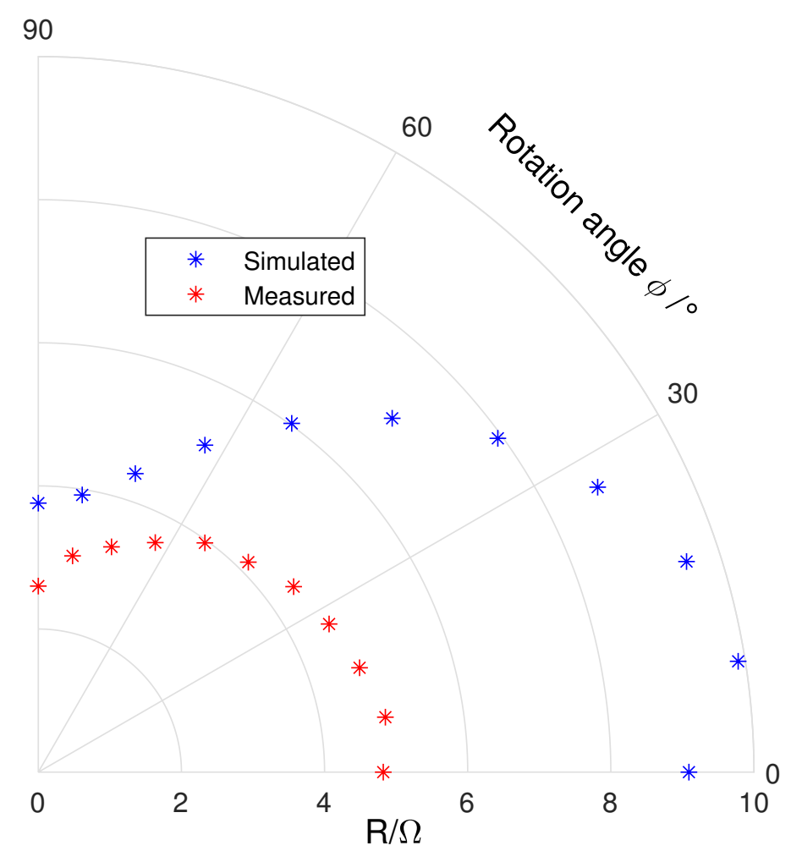


Figure 7: Measured and simulated resistances of the eleven lines, denoted by its angle.

Conclusion and Outlook

The simulation based conductivity determination has been successfully applied on meander test structures. In a next step the size of the meanders will be reduced in order to analyze the position dependency of the conductivity. In addition, further possible influences of the printing process on the conductivity (see [1]) have to be identified. In order to determine the missing σ_{zz} component an additional test structure with corresponding simulation have to be included in the determination process.

References

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